Objectives:

- Estimate function values using linear approximation.
- Compute differentials.
- Estimate error using differentials.

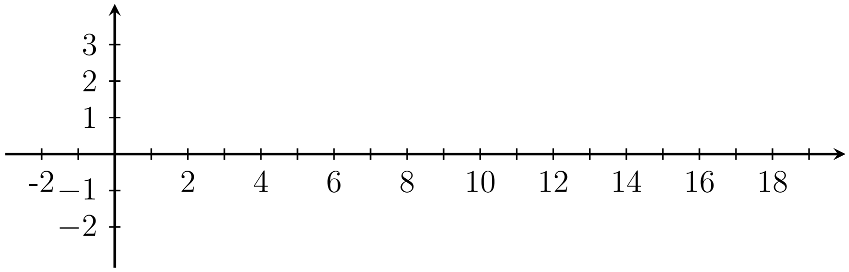
Motivation: Suppose your life depends on getting an accurate approximation of a function value but all you have at your disposal is a ruler and a four-function calculator! Or maybe you aren't given a function at all but only a nearby point and the value of the derivative at that point.

Example: Approximate $\sqrt[3]{8.1}$ using only a four-function calculator and your knowledge of calculus.

Step 1. Guess!

Step 2. Understand our estimate better using the information from the derivative.

Step 3. Use $f(x) = \sqrt[3]{x}$ and the tangent line at $(8, 2)$ to approximate $\sqrt[3]{8.1}$ better than before.



Recap: For a differentiable function $f(x)$, we can use the tangent line to $f(x)$ at $x = a$ to estimate the values of $f(x)$ when x is near a. In general, the equation of the tangent line to $f(x)$ at $x = a$ is given by

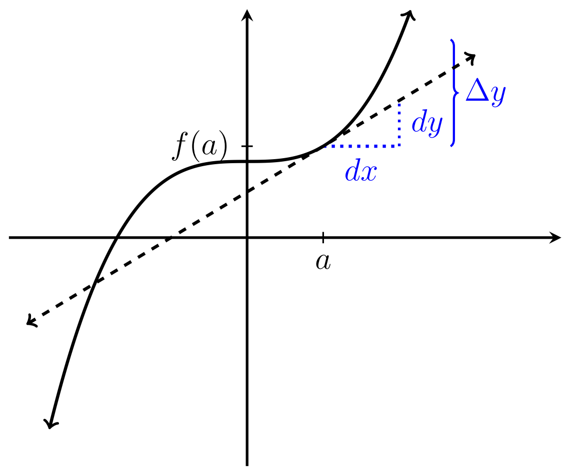
If $f(x)$ is a differentiable function at $x = a$ then the **tangent line approximation** (or **linear approximation**) to $f(x)$ at $x = a$ is the function

Near $x = a$, this can be used to approximate $f(x)$:

Example: Use the tangent line approximation to estimate $e^{0.1}$. Compare to the value given by your calculator.

If $y = f(x)$ where f is a differentiable function, then we can define the **differential** dx as an independent variable and the **differential** dy in terms of dx as

Where does this come from?



Example: Find the differential of $y = \sqrt{1 + \ln(x)}$.

Example: A cube was measured to have a side of length 20cm with an error of up to 0.3cm. What is the maximum error in measurement of the volume of the cube? How about the relative error?